Teacher's Guide

DONALD IN MATHMAGIC LAND

Subject: **Math, Science**
Level: **IJH**
ISBN: **89625-648-0**
Product: **68C82VL00**
Running Time: **26 Minutes**

Synopsis

In *Donald in Mathmagic Land*, Donald Duck gets a lesson in math appreciation when he is shown the relevance of math in everyday life. The video begins with Donald wandering into a mysterious land filled with numbers, shapes, and peculiar symbols. A narrator who calls himself "The Spirit of Adventure" informs the world's most skeptical duck that he is about to embark on a journey through the wonderland of mathematics.

Donald is whisked back to ancient Greece to meet Pythagoras, the father of math and music, and after eavesdropping on a secret meeting of Pythagoreans and turning their serene musical trio into a riotous quartet, Donald's journey continues. He takes on numerous roles, including art critic, nature observer, billiards player, baseball player, and Lewis Carroll's Alice. Through these adventures in Mathmagic Land, Donald – and viewers themselves – come to appreciate how measurements, calculations, shapes, and ratios contribute to music, architecture and art, nature, games, and inventions of all kinds, as well as how math will play a limitless role in the scientific achievements of the future.

The teacher support materials for the *Donald in Mathmagic Land* DVD are designed for educators to easily integrate segments of the movie into their daily lesson plan. The materials can be accessed by selecting the grade-level standard and expectation, which have been adapted from the *Principles and Standards for School Mathematics*[1]. From this menu, a movie clip will play that focuses on a particular math concept. Following the clip, several screens containing activity ideas are presented. Taken together, the film clips and accompanying activities facilitate a standards-aligned learning experience.

[1] Content was adapted from the *Principles and Standards for School Mathematics* (NCTM, 2000). The National Council of Teachers of Mathematics does not endorse the content or validity of these alignments.
National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics.* Reston, VA: NCTM.

PROGRAM OBJECTIVES

The student will:

- Learn how Pythagoras created the mathematical basis for our musical scale of today;
- Be introduced to the Golden Section and how it is derived from the pentagram and the Golden Rectangle;
- See how the Golden Rectangle's ideal proportions create pleasing works of art and architecture;
- Become aware of the Golden Section's occurrences in nature;
- Understand some of the ways in which mathematics is used in games and sports;
- Become acquainted with some inventions that have been inspired by imaginative uses of mathematics;
- Understand the importance of mathematics in science and technology; and
- See that the usefulness of mathematics is as limitless as our own imaginations.

PREVIEW QUESTIONS

1. What is your first thought when you hear the word "math"? Do you think math is fun? Did you ever see anything in math that surprised you?

2. Do you ever use math to solve everyday problems? How?

3. Who was Pythagoras? Who do you suppose the Pythagoreans were? Can you name any other mathematicians?

4. Can you name some ways that math is present in nature? In architecture or art? In sports or games?

POSTVIEWING QUESTIONS

1. Pythagoras was a Greek mathematician. What are some of the mathematical contributions he made?

2. What is a Golden Rectangle? Why is it important in architecture and art? Look around you right now. Do you see anything that looks like a Golden Rectangle in your classroom?

3. Pythagoras wrote, "Everything is arranged according to number and mathematical shape." What do you think he meant by this? Do you agree with him? Why or why not?

4. What are some ways that math is a part of your daily life? What about the lives of the people in your community? The government?

5. Can you suggest some ways that math is present in nature?

6. The video refers to a number of games that rely on geometric or other math principles in order to be played. What were some of these games and how is math used in them? What other games do you know of that use math principles in some way?

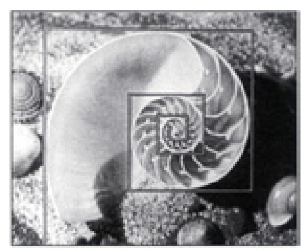
7. The video spans three millennia and shows mathematical achievements from ancient to modern times. What are some inventions that exist today that weren't shown in the video? Name some ways that math is used in these achievements.

BACKGROUND INFORMATION

The Pythagoreans
Pythagoreans were students of the mathematical, philosophical, and religious school started by Pythagoras (c. 580 B.C. – c. 500 B.C.). Some historians think that Pythagorean students were expected to listen but not contribute during their first five years at school, and that they were to credit any mathematical discovery to the school or to Pythagoras.

The Golden Section
The lines of a pentagram can be divided into four different lengths. Lines #1 and #2 exactly equal line #3. This partition is a Golden Section. And lines #2 and #3 exactly equal line #4. This partition is also a Golden Section. The ratio of the lengths of the two Golden Sections is (square root of 5 + 1)/2, approximately 1.618. When this ratio is used to create the length and width of a rectangle, the result is called a Golden Rectangle.

The Golden Rectangle
A Golden Rectangle is a rectangle whose ratio of length to width is approximately 8 to 5, or 1.618. These proportions were greatly admired by the Greeks, and Golden Rectangles are found in classical architecture and art.



The Magic Spiral
A magic spiral isn't magic at all. It is a spiral that repeats the proportions of the Golden Sections of a Golden Rectangle into infinity. Magic spirals can be seen in many of nature's spirals, such as the shell of the sea snail.

Porch of the Caryatids
The Caryatids are six female statues supporting the south porch roof of the Erechtheum temple. The temple, located on the Acropolis in Athens, Greece, was built between 420 B.C. to 406 B.C. The Caryatids that adorn the temple today are copies, but four of the six originals are housed in the Acropolis Museum.

Venus de Milo
This famous ancient Greek sculpture depicts the goddess Venus. The sculpture was named for the Greek island of Melos where it was discovered in 1820 as it was about to be crushed into mortar. The sculpture was restored, but its broken arms were lost and never replaced. The sculpture is now housed in the Louvre Museum in Paris, France.

Cathedral of Notre Dame of Paris
The Cathedral of Notre Dame in Paris, France, is regarded as the greatest masterpiece of Gothic architecture. It was constructed between 1163 and 1250 and was dedicated to Mary, the Mother of Jesus ("Notre Dame" means "Our Lady" in French). It was restored after the French Revolution ended in 1799.

Mona Lisa
This portrait of a Florentine woman was painted between the years 1503 and 1506 by Leonardo da Vinci (1452-1519). The painting was stolen from France's Louvre Museum in 1911, but was found in a Florence hotel room two years later and returned to the Louvre.

United Nations Secretariat Building
This 39-story building is one of several United Nations (U.N.) buildings located on the 18-acre U.N. complex in New York City. John D. Rockefeller Jr. donated the land and design began in 1947. The Secretariat building was completed in 1950. The U.N. Secretary General's offices are located on the 38th floor.

TEACHER SUPPORT MATERIALS

The following are teacher instructions for the activities that follow the movie segments on the DVD. Many of these activities listed for one grade level can be easily adapted for use at another grade level.

GRADES 3-5

Build a Xylophone

NCTM Standards Addressed

Measurement

- **Standard:** Apply appropriate techniques, tools, and formulas to determine measurements
    - **Grade 3-5 Expectation**: Apply standard units to measure length and volume

Connections

- **Standard:** Recognize mathematics in contexts outside of mathematics

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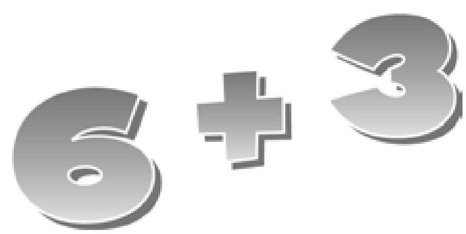
Objective: Students measure volume and experiment with musical pitch by varying the amount of water contained in several glasses.

Materials:

For each pair or group of students

- 8 glasses or jars of the same size
- Pitcher of water
- Measuring cup
- Mixing spoon
- Paper and pencil



Procedure:

1. Engage the class in a brainstorming activity by asking students: *How does music rely on mathematics?* List all possible connections between math and music on the board.

2. Show the *Donald in Mathmagic Land* segment entitled “The Mathematics of Music: The Harp.” After watching the segment, refer back to the list of math and music connections on the board. Revise the list to identify connections mentioned in the movie.

3. If you have access to an actual xylophone, share the instrument with students. Explain that they will build a xylophone out of glasses of water and a spoon. Challenge students to think about how they can achieve variations in tone and pitch with these materials. Show students the activity screen “Build a Xylophone.” It may be necessary to draw students’ attention to the image of glasses, with varied amounts of water, to help them understand how to change the pitch.

4. Assemble students into groups of two or three. Distribute the materials to each group.

5. Instruct students to experiment with how varying the volume of water contained in the glass affects the pitch produced when tapped with a spoon. Take a moment to model how they should gently strike the glass. Emphasize the importance of being consistent with where and how the glass is struck. To provide more structure, require students to add 30 milliliters or one ounce at a time and then test the sound. Have them continue to add water and test until the glass or jar is nearly full. Direct students to record the water volumes in a data table. It may be helpful to provide an example of a data table:

> **Teacher Tip**
> Like a conductor, establish a signal that identifies when the children should play/make noise and when to stop. Practice this skill with your students.

Volume of Water	Description of Sound

6. After students have experimented, require them to write a conclusion statement at the bottom of their data table that answers the question: *How does the amount of water affect the pitch?*

7. Ask students to review their data table and select 4 to 6 volumes/sounds for their xylophone (the exact amount will depend on the number of glasses available to each group). Students should note their data table selections with a * or highlighter. Then, instruct students to make their water instruments.

8. Groups can share their instruments by performing briefly for the rest of the class. After all groups have performed, facilitate a class discussion by asking such questions as:
   - How does the amount of water affect the pitch? [*Possible answer: More water produces a lower pitch*]
   - What other factors could affect the sound? [*Possible answers: the thickness of the glass, the force of the spoon tap, where the glass is tapped, the material of the spoon]*

Create a Pan Flute

NCTM Standards Addressed

Measurement

- **Standard:** Apply appropriate techniques, tools, and formulas to determine measurements
  - **Grade 3-5 Expectation**: Apply standard units to measure length and volume

Connections

- **Standard:** Recognize mathematics in contexts outside of mathematics

Objective: Students measure length and experiment with the sounds produced by blowing into different-sized straws. Once students determine how varying each straw's length affects the sound, they will build an instrument that is capable of producing multiple pitches.

Materials:

For each student

- 6 to 8 plastic drinking straws
- Scissors
- Metric ruler
- Paper and pencil
- Masking tape (a couple of rolls for the class to share)

Procedure:

1. Engage the class in a brainstorming activity by asking students: *How does music rely on mathematics?* List all possible connections between math and music on the board.

2. Show the *Donald in Mathmagic Land* segment entitled "The Mathematics of Music: The Harp." After watching the segment, refer back to the list of math and music connections on the board. Revise the list to identify connections mentioned in the movie.

3. If you have access to an actual panflute, share the instrument with students. As an alternative, you could share images and audio clips of the panflute from the Internet (see Resources section). Explain that each student will build a panflute out of drinking straws and masking tape. Challenge students to think about how they can achieve variations in tone and pitch with these materials. Show students the activity screen "Experiment with Straw Sounds." It may be necessary to draw students' attention to the image of straws and scissors to help them understand how to change the pitch incrementally.

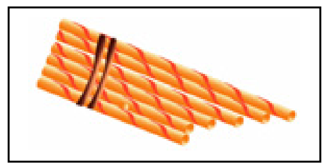
4. After distributing the materials to each student, take a moment to show them how to hold the straw against their bottom lip and blow to produce a sound.

5. Instruct students to experiment with how varying the length of the straw affects the sound produced. To provide more structure, direct students to cut 3 millimeters from the straw at a time and then test the sound. Explain that some sounds will be unclear or too similar. Whenever they come across a pleasing and distinct sound, ask them to record the length of the straw and describe the sound. It may be helpful to provide an example of a data table:

Length of Straw	Description of Sound

6. After students have recorded 6 to 8 clear sounds, require them to write a conclusion statement at the bottom of their data table that answers the question: *How does the length of the straw affect the sound?*

7. Show the activity screen "Create a Panflute." Instruct students to make their panflutes by cutting each straw to match each length recorded in their data tables. Then, they should lay the straws side by side in order from longest to shortest. Before taping the straws together, remind them to check that all of the tops are aligned. Have students place a band of masking tape around the straws to hold them together and flat.



8. Allow some time for students to play their panflutes. Then, facilitate a class discussion by asking such questions as:

   - How does the length of the straws affect the pitch? [*Possible answer: The shorter the straw, the higher the pitch*]
   - How does this musical experiment compare with the harp from the movie? [*Possible answer: The shorter string, like the shorter straw, produced the highest pitch.]*

If your students did the xylophone activity, you could ask:

- What is similar about the length of the harp string in the movie, the length of straws in the panflute, and the amount of water in the water xylophone in relation to the pitch (high or low sound)? [*Possible answer: Students might notice the longer the straw, the fuller the glass, and the longer the string produced in a lower pitch.]*

Relate this to a study of musical instruments in an orchestra. In the brass section for example, the trumpet has a higher pitch range and is much smaller than the tuba, which has a lower pitch range.

Additional Suggested Activity:

If your school has access to musical instruments, let students experiment carefully with a stringed instrument like an acoustic guitar. Tune two of the strings to make the same note. In the middle of one string place your finger lightly and allow students to plink the two strings. Do the notes harmonize (sound good to their ears)? Then place your finger at a point that is 1/3 the length of the string; 1/4; 1/5; 2/5; 3/5; and 4/5. How do the notes sound together?

Resources:
http://pan-flute.com/index_flash.html
See, hear, and learn the history of the panflute by visiting this Web site.

http://www.musicinventions.org/
The Virtual Museum of Music Invention offers helpful "Tips for Teachers" featuring an engaging interdisciplinary unit on sound with a culminating project in which students create their own musical instruments. Images and audio clips of student-created instruments are available in the online gallery.

http://www.pbs.org/teachersource/mathline/concepts/music.shtm
PBS Mathline presents a series of lessons, ranging from grades 4 through 10, that explore the connection between music and mathematics.

The Golden Rectangle Frame

NCTM Standards Addressed

Measurement

- **Standard:** Understand measurable attributes of objects and the units, systems, and processes of measurement
    - **Grade 3-5 Expectation:** Explore perimeter and area changes of two-dimensional shapes

Connections

- **Standard:** Recognize mathematics in contexts outside of mathematics

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Objective: Beginning with a square and a trapezoid, students construct a Golden Rectangle and search for the Golden proportions in their everyday surroundings.

Materials:

For each student

- A piece of cardstock
- Two index cards
- Metric ruler
- Scissors
- Red crayon, pencil, or marker
- A piece of unlined paper or index card with the dimensions 5" x 8" inches (or 12.7 cm x 20.3 cm)
- Magazines and/or picture books

Procedure:

1. Exhibit three different rectangular shapes either as drawings on the board or cardboard cutouts for display. One rectangle could have nearly square-shaped dimensions such as 16" x 14" (41cm x 36 cm). Another could be elongated with dimensions 16" x 5" (41 cm x 12 cm). And finally, a Golden Rectangle with proportions 16" x 10" (41 cm x 25 cm). Conduct a class poll to see which rectangle students like best or which rectangle seems most familiar.

2. Show the movie clip “The Golden Rectangle in Art and Architecture.” After viewing, return to the displayed rectangles. Ask students which rectangle looks like it has the proportions of the Golden Rectangle.

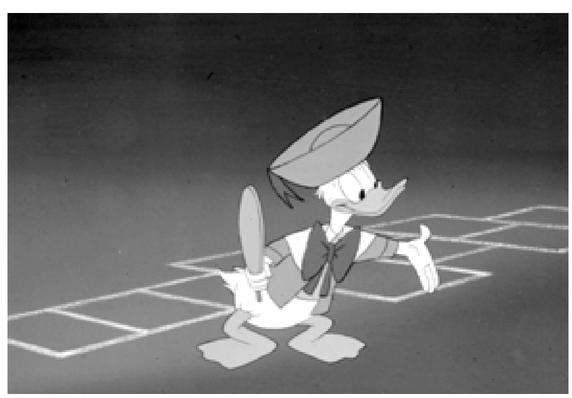
3. Display the first activity screen for “The Golden Rectangle.” While the film features the prevalence of these golden proportions in ancient art and architecture, explain to students that they will create a Golden Rectangle frame and then investigate whether this classic shape is still found today.

4. Distribute materials for making a Golden Rectangle frame and play the animated instructions on the DVD. It may be necessary to model the procedure for students and have some completed frames available as examples.

5. After students have created their own individual Golden Rectangle frames, draw their attention to the second activity screen. Explain to students that they will have some time to explore around the room to search for things that fit inside their frame’s field of view. Show students that they may turn their frames to be vertical (portrait) or horizontal (landscape). Demonstrate how to move forward and backward to make objects fit in the Golden Rectangle frame. Provide magazines and picture books for students to use as well.

6. At the end of the exploration period, redirect students to the third activity screen, “Make a Picture!” Distribute pre-cut pieces of unlined paper or index cards with Golden-Rectangle dimensions (e.g., 5” x 8” or 12.7 cm x 20.3 cm). Ask students to draw one of the things they found that fit into their Golden Rectangle frame. Show students how they can orient their picture either horizontally or vertically.

7. Provide time for students to share their pictures with the rest of the class. Facilitate a discussion with such questions as:
   - How many things can you find that fit quite well into the Golden Rectangle?
   - What other rectangular proportions did you find?
   - Why do you think the Golden Rectangle is still popular (or not as popular) today as it was during ancient times?



Additional Suggested Activity:

The video showed many examples of how mathematical logic is present in nature. Have students go on a scavenger hunt at school, at home, or in the community equipped with meter sticks. They should collect objects in nature or pictures from magazines that contain the shape of the pentagram, the magic spiral, the Golden Ratio (a ratio of length to width of approximately 8 to 5), or interesting patterns. Provide a large bulletin board on which students can create a giant collage with their pictures. Display their objects by category.

Head-to-Height Proportion

NCTM Standards Addressed
Measurement
- **Standard:** Apply appropriate techniques, tools, and formulas to determine measurements
  - **Grade 3-5 Expectation:** Select and use benchmarks to estimate measurements

Connections
- **Standard:** Recognize mathematics in contexts outside of mathematics

Objective: Students determine average head-to-height proportions for the class by using nonstandard measurements and creating graphs.

Materials:

For each pair of students
- String or yarn (about one meter or one yard in length)
- Scissors
- Graph paper and pencil
- Cartoon examples (Optional)

Procedure:

1. Begin the lesson with a class discussion about the concept of proportion. You may want to show some political cartoons in which facial body parts (e.g., nose, ears) are out of proportion. Ask students if they have ever looked at themselves in a fun-house mirror. Prompt them to explain why looking at themselves in a fun-house mirror is funny. Students may respond with examples such as their head or legs were too long to fit with the rest of their body. Use this prior knowledge to help students understand proportion—the relation of parts to each other or to the whole.

2. Introduce the *Donald in Mathmagic Land* movie clip by explaining that students will see examples of a specific proportion that can be found in art, architecture, and even natural structures such as the human body. Show the movie clip "The Golden Rectangle in Art and Architecture."

3. In the movie, Donald did not quite fit the Golden proportion. Show the activity screen "Measuring Donald." From the image, ask students to figure out Donald's head-to-height proportion by asking: *How tall is Donald if you measured him in duck heads?* Students should estimate Donald's height to be 3 1/3 or 3 1/4 duck heads.

4. Display the second activity screen "Measuring a Partner." Explain to students that they will use a string to first measure the length of a partner's head (from the top of the head to the bottom of the chin) and then the partner's height using the same piece of string. Demonstrate how to perform the measurement and emphasize the safe handling of scissors. Once students have measured the head-length piece of string, they should move away from their partner before cutting the string with scissors. After measuring their partner's height with the head-length string, students should switch positions and repeat the

> **Teacher Tip**
> Children will get a more accurate measurement of their height if they use a flat surface when measuring lengths. They could mark their height on the wall (with masking tape) or by lying down while the partner measures with the string.

process using a new piece of string. You may want to ask students to consider why it would be important to use a new head-length string as a standard of measurement in determining proportion.

5. Allow time for partners to measure each other. Ask students to record both of their names and head-length measurements on a piece of paper.

6. Then direct students to find out the head-length heights of the rest of their classmates. Depending on class size, you may want students to survey all of their classmates or a smaller group of 5 to 10 students.

7. Show the final activity screen "Graphing." Instruct students to create a bar graph that shows each students' height in head lengths.

8. Instruct students to determine the average height in head-length units. You may choose to have them visually estimate the average using the bar graph or calculate the average height using the recorded measurements.

9. Conclude by having students share and compare averages. For the whole class, determine the proportion and express it as a ratio of height in head lengths to one head. You may want to indicate that the height of the average adult is seven head lengths (7 to 1).

Resources:
http://www.figurethis.org/challenges/c61/challenge.htm
This activity asks students to determine if the Statue of Liberty's nose is out of proportion to her body size. The activity, from the *Figure This!* list of 80 math challenges, illustrates how to use similarity and scaling to design HO gauge model train layouts and analyze the size of characters in Gulliver's Travels.

http://www.worsleyschool.net/socialarts/body/proportions.html
The activity above was adapted from ideas presented in "Body Proportions in Art." Review this Web site for more instructional activities.

Geometric Shapes in Multiple Dimensions

NCTM Standards Addressed

Geometry

- **Standard:** Use visualization, spatial reasoning, and geometric modeling to solve problems
  - **Grade 3-5 Expectations:** Build three-dimensional geometric objects and draw two-dimensional representations

Connections

- **Standard:** Recognize mathematics in contexts outside of mathematics

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Objective: Students construct a series of three-dimensional shapes, predict, and then create the two-dimensional cross sections for each shape.

Materials:

For each individual or pair of students

- Clay (a 5-centimeter diameter clump of clay should be a sufficient amount)
- Thin string, dental floss, or fishing line (approximately a foot-long or thirty centimeters)
- Paper and pencil

Procedure

1. Provide samples of objects in both two and three dimensions. For instance, two-dimensional pictures of fruit or flowers as well as the actual objects. By comparing the pictures and the objects themselves, ask students to define what makes something three-dimensional.
   As another option, if you happen to have three-dimensional pictures and glasses, students could observe the pictures with and without the glasses to see the difference between two- and three-dimensions.

2. Introduce the segment "Spinning Circles" from *Donald in Mathmagic Land* by explaining that they will see shapes change from 2-dimensional to 3-dimensional and vice versa. Play the movie clip.

3. For the post-viewing activity, students may work individually or in pairs. Distribute the pieces of clay and string to each individual or pair.

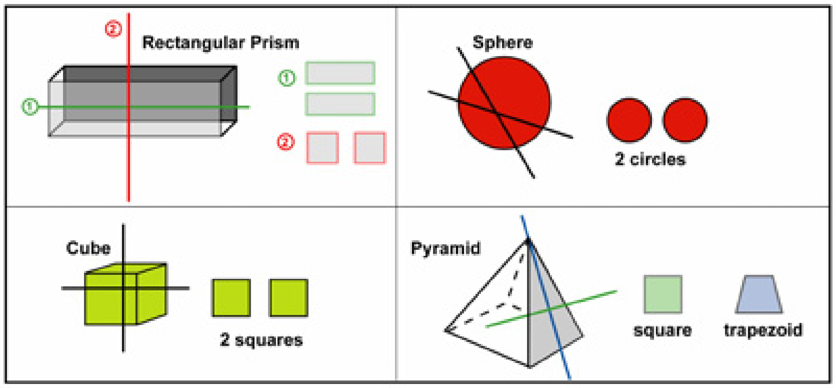
4. Show the first activity slide, "Create a 3-dimensional Rectangle." Give students a moment to mold their clay into a rectangular prism (2-dimensional shape that has height) like the one shown on the screen.

5. Instruct students to imagine they have made a cross-sectional slice into their prism. It can be either a vertical or a horizontal slice. What do they envision the cross section would look like once they separated the two pieces and see the inside? Ask students to draw and label their predicted shapes on a piece of paper. You may want to have students record their predictions in a 3-column table with the following headings:

3-D Shape	Pre-slice Prediction	Actual 2-D Shapes (post-slice)
Draw and label the constructed shape	Draw and label the predicted shapes of the cross section	Draw and label the resulting cross-sectional shapes

6. Before advancing to the next activity screen "From 3-Dimensional to 2-Dimensional," call on a few students to share their predictions. Then, direct students to use their string to make one slice in their prism. It can be either a vertical or horizontal cross section. Ask students to draw and label the actual shapes. Were their predictions correct?

7. Display the final activity screen, "Spheres, Cubes, and Pyramids." Instruct students to repeat the steps—mold, predict, and slice—for each of the shapes. Again, they should record their predictions and results through drawings and labels. Correct answers include:



8. As a culminating discussion, remind students that the movie demonstrated how a cross-sectional slice from a sphere provided a lens-shape used in magnifying glasses and microscopes. Have students review the resulting shapes from their cross-sectional slices. Are these shapes that can be found in structures and technology? Ask students to share some examples of applications.

Resources:

http://illuminations.nctm.org/index_d.aspx?id=406
Elementary-aged students can explore geometric solids using a series of interactive lessons available through the National Council of Teachers of Mathematics' Web site.

http://matti.usu.edu/nlvm/nav/frames_asid_126_g_3_t_3.html?open=instructions
This online manipulative allows students to explore several different platonic solids by virtually slicing through them.

Mathematics in Sports and Games

NCTM Standards Addressed

Geometry

- **Standard:** Use visualization, spatial reasoning, and geometric modeling to solve problems
  - **Grade 3-5 Expectations:**
    - Recognize geometric ideas and relationships in other disciplines
    - Identify and compare attributes of two- and three-dimensional shapes

Objective: Students measure, identify, and compare different geometric shapes found in sports and games.

Materials:

- Assortment of sports equipment for the class to share (several suggestions provided in activity screen "Comparing Sports Equipment")
- String (one-meter- [yard-] long piece of string per student or pair)
- Metric ruler or metric measuring tape
- Paper and pencil

Procedure:

1. Begin with a small group or whole class brainstorming activity called List-Group-Label (Barton & Heidema, 2002). Have students think about a favorite sport or game. Instruct students to list how mathematics is used in that particular sport or game. The list should consist of single words or brief phrases. Ask students to review the list and organize the words and phrases into categories. Once students have classified their list, ask them to explain the rationale behind their groupings by writing a label for each category. For instance, categories may reflect how mathematics functions in the sport or game such as equipment, setting, strategy, and score keeping. Students may group according to the mathematical content or skill: such as arithmetic, geometry, problem solving, or measurement.

2. Show the movie clip "Mathematics in Sports: Shapes" from *Donald in Mathmagic Land.* Tell students they may add to their list if they find a new mathematical connection in the movie.

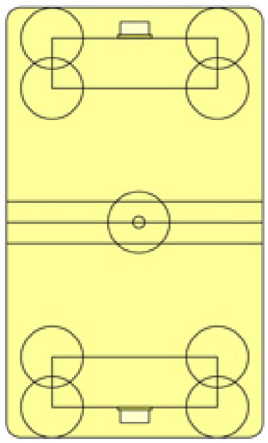
3. The activity screens that accompany this movie segment allow for instructional flexibility. Display the first activity screen "Comparing Sports Equipment." You will need to have a selection of sports equipment available in class for students to measure, as required by the last two tasks outlined on the activity screen. You may choose to have students work independently or in pairs to respond in writing to all four tasks. As an alternative, you could engage the class in a discussion of the first three tasks using either the images on the screen or the actual sports equipment.

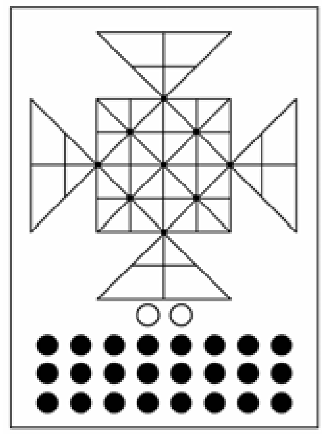
4. The third and fourth tasks on the activity screen can be developed to suit your grade-level and curricular needs. For instance, to measure the objects, you could focus on mass, length, width, circumference, perimeter, area, and/or angles.

> **Teacher Tip**
> Be sure to clearly indicate how to identify and count the shapes and lines. For instance, will a pair of perpendicular lines count once or will each line be counted? Will each congruent triangle count once or each time it appears?

5. After you have stated your measurement expectations, provide each student or pair with a piece of string and ruler. Model how to use these tools for measuring the objects as well as how to record measurements.

6. The next two activity screens, "Comparing Sports Settings" and "Game Board Shapes," can be completed orally as a class or individually in writing. Allow time to share answers as a class. Answers to questions will vary according to you students' attention to detail. You could turn this activity into a class competition among individuals or pairs.





Additional Suggested Activity:

Ask students to select their favorite sport or hobby and think of the ways that math is used in it (scoring, measuring, distance and time, averaging, etc.). Provide time for students to share their sport or hobby and the mathematical principles in it through demonstrations, pictures, props, or multimedia presentations.

Resources:

Internet

http://matti.usu.edu/nlvm/nav/frames_asid_165_g_1_t_3.html?open=instructions
This virtual manipulative allows students to construct their own congruent triangles.

http://oncampus.richmond.edu/academics/education/projects/webunits/math/sport.html
Mathletics, developed by a preservice teacher at the University of Richmond, presents a series of math activities in the context of sports.

http://www.shodor.org/interactivate/activities/index.html
Project Interactivate, from The Shodor Education Foundation, offers an extensive collection of Java applet online interactives that explore statistics and probability concepts. From a dice game, to the stock exchange, to a game show, students can learn about probability in a variety of contexts.

Print

Barton, M. & Heidema, C. (2002). *Teaching Reading in Mathematics* (2nd Ed.). Aurora, CO: McREL.

Sierra, J. & Kaminski, R. (1995). *Children's Traditional Games: Games from 137 Countries and Cultures*. Phoenix, AZ: Oryx Press.

GRADES 6-8

Angles

NCTM Standards Addressed

Measurement

- **Standard:** Apply appropriate techniques, tools, and formulas to determine measurements
  - **Grades 6-8 Expectation:** Apply techniques and tools to accurately measure angles

Geometry

- **Standard:** Apply transformations and use symmetry to analyze mathematical situations
  - **Grade 6-8 Expectation:** Examine line symmetry of objects using transformations

Objective: Students will use a protractor to measure the angle of incidence and angle of reflection.

Materials:

For each group of students

- Protractor
- Projector/light source (LCD projector or laser pointer)
- Full length mirror
- Chalk board erasers and chalk dust (or talcum powder)
- Graph paper

Before you begin:

1. The following is an optional demonstration that shows the angle of incidence and the angle of reflection using a beam of light. The demonstration uses chalk dust. Before you begin, ask if there is anyone in the room who is allergic to chalk dust and if so, you may want to have them wait outside or go to the library to complete an alternate assignment for the first part of the lesson.

2. Place the mirror in a location where all the students can see it. Set up the projector such that a beam of light can be reflected at different angles (you may want to place the projector on a cart).

Alternate Strategy Tip

Have students experiment themselves by using a flashlight. If you choose to do this, students should tape over the flashlight so that just a slit is open for light to come thorough (this will make it easier to see the light beam). Students can draw angles of incidence and reflection, measure them, and come to their own conclusions about these angles.

Procedure:

1. Turn off the lights in your classroom and close the blinds to make it as dark as possible. Turn on the projector so that the light strikes the mirror. Ask students to make observations about the light that hits the mirror. Students should notice that the light is reflected and may see the reflected light on a wall of the classroom. Caution students not to look directly into the light.

2. Move the projector so that the angle at which the light hits the mirror changes. Ask students to describe what happened to the reflected light when the projector moved.

3. Create beams of light by hitting two chalkboard erasers together to create dust. Students should notice that the light reflects off the dust particles creating a light beam. As you do this explain to students that the light beam that originates from the projector and strikes the mirror is called the **ray of incidence**, while the beam that reflects from the mirror is called the **ray of reflection**. You may want to have the students draw a labeled picture of this in their notebook or journal.

4. Next, demonstrate the normal line by using a meter stick. While the beams are illuminated, place the meter stick perpendicular to the mirror. Explain to students that this is the normal line. Show students the angle from the light beams to the mirror's surface. The angle from ray of incidence to the mirror is called the **angle of incidence**. Likewise the angle from the ray of reflection to the mirror is called the **angle of reflection**. Move the projector so that the angles change. Ask students to note what happens to the angle of reflection when the angle of incidence is increased (the angle of reflection also increases).

5. Show the *Donald in Mathmagic Land* clip "Mathematics in Games: Billiards." When the clip is complete, refer to the first activity screen "Billiard Angles." Ask students to apply what they learned about light with what they saw on the movie clip. Students should be able to relate the similarity between the ball hitting the cushion and the light striking the mirror. Review the angles shown on the "It's All in the Angle" activity screen.

6. The "Play Paper Billiards" activity screen describes a fun way for the students to draw angles on graph paper. Encourage students to complete the two suggested experiments. Circulate around the room, providing assistance with the use of the protractor. Ask questions to gauge students' understanding of different angles.

Resources:

Internet

http://genesismission.jpl.nasa.gov/educate/scimodule/CollProcess/CollProcess_pdf/ConcentratorST.pdf Student Text on Mirrors, Parabolas, and the Genesis Concentrator

http://illumtest.nctm.org/imath/6-8/pooltable/pool1.html
The National Council of Teachers of Mathematics' Web site features a paper pool game with instructions and an online interactive.

http://matti.usu.edu/nlvm/nav/frames_asid_284_g_3_t_3.html
Using this online manipulative, students can create and measure angles on a circular geoboard.

http://www.shodor.org/interactivate/activities/angles/index.html
Students can test their understanding of acute, obtuse, and alternate angles with this online quiz.

Print

Smoothey, M. (1993). *Let's Investigate Angles.* North Bellmore, New York: Marshall Cavendish Corporation.

The Pythagorean Theorem

NCTM Standards Addressed

Geometry

- **Standard:** Analyze characteristics and properties of two- and three- dimensional geometric shapes and develop mathematical arguments about geometric relationships
  - **Grade 6-8 Expectation:** Create deductive arguments concerning the Pythagorean relationship

Objective: Students will prove the Pythagorean Theorem geometrically.

Materials:

For each group of two or three students

- One centimeter dot paper, See PDF on this DVD
- Geoboard with rubber bands (Optional)
- Ruler

Procedure:

1. Show the short clip from *Donald in Mathmagic Land* "The Pythagorean Theorem." As the movie shows the writing on the wall, point out the diagram representing the Pythagorean Theorem.

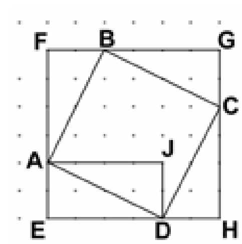
2. Provide the following information to your students: *In algebraic terms the Pythagorean Theorem states that* $a^2 + b^2 = c^2$ *where c is the hypotenuse while a and b are the legs of the triangle.*

3. Distribute the dot paper to your students. Show the first activity screen, "The Pythagorean Theorem" to your students and instruct them to follow the directions. When ready, show students the second screen. Ask students to answer the last question on this screen.

4. Tell students that if they rotate and flip the entire drawing so the hypotenuse is on the bottom, they will see an image similar to the one that was used on a Greek stamp issued in 1955 to commemorate the 2500th anniversary of the Pythagorean School. See the reference section for a Web site that shows this stamp.

5. For the third activity screen, students are given another triangle that is a bit harder to solve. Students may need some assistance on this one! The following is one way to solve it:

   a. Measure the distance of the hypotenuse.
   b. Use this distance to make a square with the hypotenuse as one of the sides. (A, B, C, D)
   c. From the original triangle, make a rectangle by drawing another triangle that shares the hypotenuse (A, J, D, E). Determine the area of this rectangle (8).
   d. Draw a large square (E, F, G, H) that connects all of the vertices.
   e. Find the area of each of the triangles that are inside the square (E, F, G, H) and outside the square (A, B, C, D).
   f. There should be four triangles each with an area of four. (Δ A,E,D), (Δ A,F,B), (Δ B,G,C), (Δ C,D,H), 4 x 4 = 16
   g. Subtract the area of EFGH (36 cm^2) minus the area of the four triangles (16 cm^2) to come up with the answer of 20 cm^2.



Resources:
http://matti.usu.edu/nlvm/nav/frames_asid_129_g_1_t_3.html?open=activities
The National Library of Virtual Manipulatives features an online geoboard students can use to create right triangles and other shapes.

http://matti.usu.edu/nlvm/nav/frames_asid_164_g_3_t_3.html?open=instructions
This virtual manipulative enables the student to move around squares and triangles to demonstrate the validity of the Pythagorean Theorem.

http://www.ies.co.jp/math/java/geo/pythagoras.html
This Internet site offers 19 manipulative applets designed to help students acquire an intuitive understanding of fundamental concepts of the Pythagorean Theorem.

Patterns in Polygon Sides and Diagonals

NCTM Standards Addressed

Algebra
- **Standard:** Understand patterns, relations, and functions
  - **Grade 6-8 Expectation:** Represent, analyze, and generalize patterns with tables

Geometry
- **Standard:** Use visualization, spatial reasoning, and geometric modeling to solve problems
  - **Grade 6-8 Expectation:** Use geometric models to represent algebraic relationships

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Objective: Students investigate a series of polygons to determine a pattern between the number of sides and the number of diagonals for each geometric shape.

Materials:
- Paper and pencil

Procedure:
1. Write several Greek and Latin-based numeral prefixes on the board such as: uni-, mono-, bi-, duo-, tri-, quadi-, pent-, quint-, hex-, sext-, hept-, sest-, oct-, nonus-, deca-, etc. Be sure to scramble up the prefixes. See if students can identify the numbers associated with each prefix. Extend the discussion by asking students to think of examples of words that use these prefixes.

2. Explain to students that they will watch a segment from *Donald in Mathmagic Land* that shows an “Infinite Pentagram.” After the introductory activity, students should readily identify that the pentagram will involve the number five. Challenge students to identify something else shown in the movie clip that involves the number five, other than the pentagram. Some students may notice the pentagon shape at the center of a pentagram.

3. After the movie, show the first activity screen "Pentagon and Pentagram." Go over the examples of polygons shown on the screen.

4. Advance to the second activity screen "Polygon Sides vs. Diagonals." Ask students to create a chart on their own paper like the following:

Shape	Number of sides	Number of diagonals
Triangle	3	0
Quadrilateral	4	2
Pentagon		
Hexagon		
Heptagon		
Octagon		
Nonagon		
Decagon		

5. As students complete the table, instruct them to draw each shape and insert diagonals between the vertices. This will enable them to identify and count the number of diagonals for each polygon.

6. When the table is complete, direct students to write a statement explaining the relationship they have discovered between the number of sides and the number of diagonals in the polygons. Students should identify that as the number of sides increases, the number of diagonals also increases. Encourage students to tackle the "Challenge" on the activity screen. You may need to provide some hints to help students determine the algebraic equation:

$$\mathbf{\frac{n\,(n-3)}{2} = \text{number of diagonals}}$$

** n represents the number of sides in a given polygon*

Teacher Tip
There are several other formulas that students might find. They are all equivalent to this one. Students should be encouraged to explain why this formula (or others that they find) work. For example, they should be able to explain why dividing by 2 is necessary (using their pictures). They should be able to explain why they subtract 3 from n, and why they multiplied n times (n-3).

Additional Suggested Activity:
The equal-sided pentagram was the Pythagoreans' emblem. Provide protractors and pencils and allow students to experiment with the shape's construction, or share these steps: (1) Use a pencil to draw a circle with a lid or similar object on a piece of paper and mark the center of the circle, (2) Locate five equally spaced points (one for each tip of the pentagram) around the circle using a protractor. *(Hint: A complete circle is 360 degrees. If the five points are equally spaced around the circle, then each arc of the circle is 360/5 = 72 degrees. Draw a point at the top of the circle and use the protractor to mark 72 degrees around the circle until you come back to the point at the top.),* (3) Erase the circle and its center and all the construction lines, and then join the five points as shown.

Resources:
http://www.intermath-uga.gatech.edu/tweb/CPTM1/trushin/diagonals/diagonalsinapolygon.htm
This site features a similar math challenge. Students are asked to determine a pattern between the number of sides and diagonals in polygons and apply this pattern to predict the number of diagonals in a dodecagon (12-sided polygon). The explanation provided may be helpful to refer to or share with students.

http://www.cut-the-knot.org/Curriculum/Geometry/DiagonalCount.shtml
Students can use this online interactive to draw and count diagonals in a series of polygons. The challenge, however, is that the diagonal lines may not cross.

The Surface Area of 3-D Shapes

NCTM Standards Addressed
Measurement
- **Standard:** Understand measurable attributes of objects and the units, systems, and processes of measurement
    - **Grade 6-8 Expectation:** Use units to measure area and surface area

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Objective: Students study the applications of cylinders and shapes and learn to calculate the surface area of a few 3-dimensional shapes.

Materials:
- Samples of cylindrical-shaped household items (e.g., a spool of thread or kite string, a fishing reel, a roll of film, toilet paper/paper towel tube, rolling pin, can of soup, etc.)
- Several different-sized paint rollers with handles
- Paint
- Paint trays
- Large pieces of paper (e.g., butcher paper or newspaper)
- Metric rulers
- Paper and pencil

Procedure:
1. Display several cylindrical-shaped household items. Some examples are: a spool of thread or kite string, a fishing reel, a roll of film, toilet paper/paper towel tube, rolling pin, etc. Facilitate a class discussion by asking students to consider the function and practicality of the cylinder shape for each of the household items. Would the items be able to serve the same function if they were shaped differently?

2. Explain to students that they will watch a segment from the movie *Donald in Mathmagic Land* that shows some of the many uses of cylinders and other geometric shapes. Show the movie clip "Applications of Cylinders and Shapes."

3. After watching the film, challenge students to think of other possible applications.

4. Display the first activity screen "How do you determine the surface area of 3-dimensional shapes? Show students how to apply the area formula a = l x w or $a = s^2$ to determine the surface area of a 3-dimensional cube. If calculating the area of a quadrilateral is relatively new to your students, it may be necessary to provide additional examples before moving on to the more challenging cylinder shape. Also, if students are unfamiliar with finding the area of circles, you may want to guide them through some examples.

5. Display the next activity screen, "How do you determine the surface area of three-dimensional shapes?" At first students may not recognize the relevance of the rectangular shape. Spend some time discussing the various two-dimensional shapes that comprise the three-dimensional cylinder. Using the cylinder pictured, ask students to explain—in writing or orally—how they plan to figure out the surface area of the cylinder. Discourage them from making any calculations yet. At this point, they should only describe the procedure they will use to solve the problem. Then, instruct students to calculate the surface area. After figuring out the area of the top, bottom, and middle, they should add the areas together and come up with 88.84 cm^2.

6. Advance to the "Paint Roller Challenge" screen. Students may work either individually or with a partner. To assist with managing the materials, you may want to set up several different stations, each numbered with a different-sized roller, paint tray, and metric ruler. Model for students how to roll the paint onto a large piece of paper. Emphasize the importance of only completing one full rotation of the paint roller. Then, students should measure the length and width of the rectangular paint mark and calculate the area. Direct students to label their paint mark with the designated station number. This will avoid confusion when they rotate to other stations.

7. Instruct students to be careful as they carry their butcher paper or newspaper to the next station. Have students rotate through several different stations.

8. After students have rotated through the stations, ask them to compare the surface areas of each paint roller. Ask them to select the paint roller that would be most efficient for painting a room. Then prompt students to write a recommendation in which they explain why they feel their selected roller is the most efficient. Have students share their recommendations with their classmates.

Resources:

http://www.shodor.org/interactivate/activities/sa_volume/index.html
In this online interactive developed by *Project Interactivate*, students experiment with surface area and volume as they manipulate the dimensions of a polyhedra. Suggested discussion and exploration questions as well as a student worksheet are all provided.

GRADES 9-12

Conic Sections

NCTM Standards Addressed

Geometry

- **Standard:** Use visualization, spatial reasoning, and geometric modeling to solve problems
    - **Grade 9-12 Expectation:** Analyze the cross sections of three-dimensional objects

Objective: Students will determine the angle and location of the intersection of a plane through a cone, then describe how the angle and location of the slice affect the section.

Materials:

For each group of students

- 4 Polystyrene cones, 3" x 6" (7.6 cm x 15.25 cm) (purchased from a craft store)
- Steak knife or similar cutting utensil
- Ruler

Procedure:

1. Have students work in groups of two or three students. Ask them to brainstorm how the full cone shapes and partial cone shapes are used in everyday life. Possible examples of full cones might include: ice cream cones, paper snow cone holders, megaphones, cones used on the highway and track, nose cone on a rocket, tee pee, and horns; partial cones include: lampshades and polystyrene cups. Ask students to list the characteristics of the cone that make it useful.

2. Introduce the phrase "conic sections" to students. Ask them to provide a definition before watching the movie clip. Show the *Donald in Mathmagic Land* clip "Conic Sections." As students watch, ask them to revise their definition based on what was shown in the clip.

3. Instruct students to read the first activity screen that follows the movie clip. Ask them to relate what is shown on the screen that is similar to or different than their revised definition.

4. Distribute the materials to each group. Have students follow the instructions on the second activity screen. Once students have made the circle, ellipse, and parabola, have them compare these cross sections by determining the following:

    - Circle: Students can measure the diameter, radius, and calculate the circumference. $C = \pi d$
    - Ellipse: Students can measure the major and minor axes, and calculate the circumference. Approximated by the equation $C = 2\pi \sqrt{(a^2 + b^2)}$; where a and b are the major and minor axes.
    - Parabola: Students can measure the distance from the vertex to the focus. Students may need assistance locating the focus of the parabola. Instructions for doing this can be found in the activity "Parabolic Problem" in the reference section below.

5. Allow students to make additional slices to their cones as long as they are parallel to their original slice. Students can then compare the shapes as described in procedure #4 above.

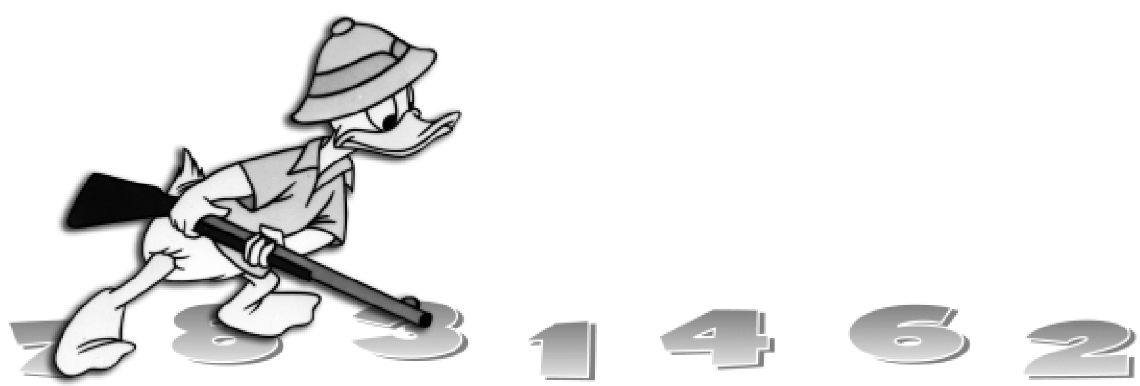
6. Show the third activity screen and instruct students to make a section that makes a hyperbola. For the questions that follow, students should describe how each section would change in size by changing the location of the slice. Based on what they learned in the physical model, for the hyperbola, students should conclude that as the slice is closer to the point where the two cones meet, the hyperbola gets larger. For the circle and ellipse, students should realize that the closer the slice is made to the point, the smaller the circle or ellipse will be. Conversely, the further away the slice is made from the point, the larger the circle or ellipse will be. As for the inclination of the angle, when the slice is horizontal, the result is a circle. As the angle increases, the resulting eccentricity of the ellipse increases until eventually a parabola and then a hyperbola form.

7. Conclude this lesson by asking students to summarize what they have learned about conic sections with an emphasis about how the resulting shapes are useful to humans.

Resources:

http://genesismission.jpl.nasa.gov/educate/scimodule/CollProcess/CollProcess_pdf/ParabolicProblemTG.pdf
Students determine the equation for a parabola and graph the parabola using measurement similar to that of the Genesis concentrator.

http://math2.org/math/algebra/conics.htm
Illustrations and definitions of conic sections with equations.



The Golden Section

NCTM Standards Addressed

Geometry

- **Standard:** Use visualization, spatial reasoning, and geometric modeling to solve problems
    - **Grade 9-12 Expectation:** Draw and construct representations of two-dimensional geometric objects

Objective: Students will construct a Golden Rectangle and draw and compare a Golden Spiral from a Golden Rectangle and a Golden Triangle.

Materials:

For each group of students

- Paper
- Metric Ruler
- Compass
- Protractor

Procedure:

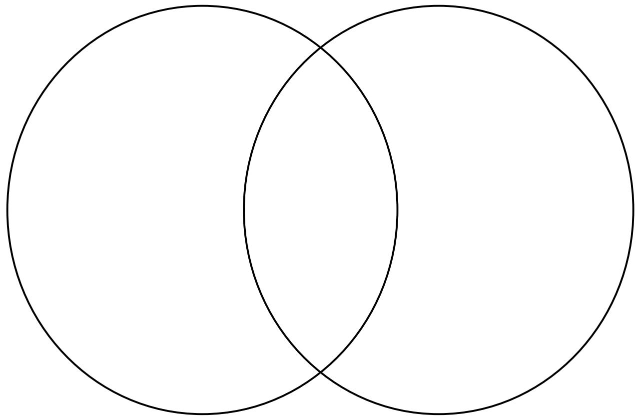
1. Show students the “Golden Section, Golden Rectangle, and Golden Spiral” clip from *Donald in Mathmagic Land*. As students watch, ask them to develop a definition of “Golden Section.” Tell students that they will be able to refine their definition, as they construct and compare Golden Spirals.

2. Explain to students that they will be constructing a Golden Rectangle. Have students use their materials and follow the directions on the first activity screen. Have students measure the length and the width to determine the proportion of the Golden Rectangle. Students should then follow the steps in the subsequent screens to create a Golden Spiral. As they do this, they will also be constructing smaller Golden Rectangles. Each time they do, have them measure the length and the width of the rectangle to determine if the proportion is the same.

3. Next, explain to students that they will be creating another Golden Spiral, this time from a Golden Triangle. The Golden Triangle is an isosceles triangle in which the ratio of the base to the side is equal to the Golden Ratio. Display the first Golden Triangle screen and ask students to use their protractor to construct a triangle with the designated angles and a 10-centimeter base. The following screen instructs students to use their protractor to bisect angles to create smaller triangles. Finally, students draw arcs to create the Golden Spiral.

4. On the last screen, students are asked to compare the Golden Spirals. One strategy for students to use for this would be for them to create a Venn Diagram with the intersection of the circles containing the similarities (both have a ratio base to side of 8:5; both are spirals; they both appear to get infinitely smaller and smaller), while the differences would be on the outside circles (the triangle spiral appears to be “tighter,” and the method that was used to construct the spirals was different).

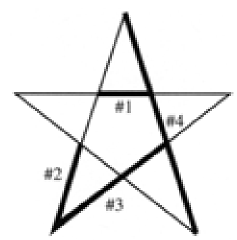
5.



Rectangle Spiral Similarities Triangle Spiral

6. Conclude the lesson by asking students to refine their definition of a Golden Section based on the movie clip and their experiences with constructing Golden Spirals. Encourage students to include the proportions as part of their answer.

Additional Suggested Activity:



Enlarge and reproduce this pentagram on page 2 and give each student a copy with a metric ruler. Use a calculator to compute the ratios of the lengths of lines #1 and #2 (divide the longer length by the shorter), the lengths of lines #2 and #3, and the lengths of lines #3 and #4. If students are careful in measuring they will get the same ratio (lines #1 and #2 exactly equal line #3, and lines #2 and #3 exactly equal line #4). Challenge students to figure out the Golden Ratio accurately using algebra. *(Let the length of line #1 be one unit and the length of line #2 be x. The Golden Ratio is the ratio of the length of #2 divided by the length of #1, which is x/1. But since the Golden Ratio is the length of #3 divided by the length of #2, and since the length of #3 is x + 1, we see that the Golden Ratio is also (x + 1)/x, so it must be that:*

$$\frac{\#2}{\#3} = \frac{\#3}{\#2} = \frac{\#2 + \#1}{\#2} \quad \text{or} \quad \frac{x}{1} = \frac{x + 1}{x}$$

That equation is equivalent to the following:

$$x^2 = x + 1 \quad \text{or } x^2 - x - 1 = 0$$

The positive solution to that equation is x = 1 + Square Root of 5)/ 2, *which is approximately 1.618034).*

Resources:

http://cuip.uchicago.edu/~dlnarain/golden/

This Internet site introduces the meaning and value of the Golden Ratio through a set of seven student activities.

http://math.rice.edu/~lanius/Geom/golden.html
The *Golden Ratio* lesson scaffolds a series of activities as students find Golden Rectangles, build them, and confirm the ratio using algebra.

http://matti.usu.edu/nlvm/nav/frames_asid_133_g_4_t_3.html?open=instructions
This virtual manipulative can help students visualize the Golden Rectangle. It shows how a set of Golden Rectangles is generated by using the Golden Ratio (the ratio of the longer side to the shorter side of a Golden Rectangle) to create smaller Golden Rectangles within an initial rectangle.

Mathematics in Nature

NCTM Standards Addressed

Geometry

- **Standard:** Use visualization, spatial reasoning, and geometric modeling to solve problems
    - **Grade 9-12 Expectation:** Use geometric ideas to gain insights into other disciplines

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Objectives: Students identify examples of symmetry in organisms and relate this to how these organisms move and obtain resources. Students list examples of fractals in nature and construct a fractal "tree."

Materials:

For each student

- Internet or resource books on organisms

Procedure:

1. Have students define the word "symmetry" using direct vocabulary instruction.
   Direct Vocabulary Instruction as described by Marzano et al.(2001):
   - Students must encounter words in context more than once to learn them.
   - Instruction in new words enhances learning those words in context.
   - One of the best ways to learn a new word is to associate an image with it.
   - Direct vocabulary instruction on words that are critical to new content produces the most powerful learning.

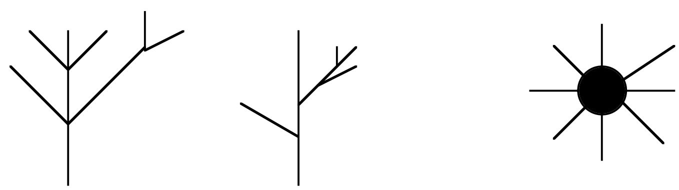
Example: Using the Glossary for Direct Vocabulary Instruction *Symmetry*

a) Present students with a brief explanation or description of the new term or phrase. For example: "Symmetry: Correspondence of opposite parts in size, shape, and position."
b) Present students with a nonlinguistic representation of the new term or phrase. Show the video clip associated with the term in "Mathematics in Nature" from *Donald in Mathmagic Land*.
c) Ask students to generate their own verbal description of "symmetry."
d) Ask students to create their own nonlinguistic representation of "symmetry."
e) Periodically ask students to review the accuracy of their explanations and representations.
f) Show the activity screen with three types of symmetry in nature. Have students repeat these steps with each new term, *spherical*, *radial*, *bilateral*, as well as *asymmetry*.

2. Provide students with resources in order to find other examples of symmetry in nature. As students research these examples, have them find out how symmetry affects the organisms' ability to move and obtain resources. As students conduct this research, they will begin to realize that organisms that have bilateral symmetry have the most efficient method of obtaining resources. Make sure that students gather evidence to support the generalizations that they make about the effects of symmetry on the organisms.

3. Conduct a direct vocabulary instruction experience for the term "fractals" using the second activity screen fractals in nature. Provide students with books or Web sites that give examples of fractals in nature with an emphasis in plants.

4. The fractal "tree" shown on the third activity screen is an example of a tree that has an opposite branching pattern. Briefly demonstrate branching patterns of other trees such as alternate and whorled. Ask students if they would consider if these other branching patterns are fractals based on their working definition. Have students relate these branching patterns to the type of symmetry on the previous screens.



Opposite Alternate Whorled (top view)

5. Challenge students to create an extensive fractal pattern for the tree as illustrated on screen three.

6. Optional: Have students conduct research about how fractals in nature can be mimicked by humans as they design cities.

Resources:

Internet

http://illuminations.nctm.org/mathlets/fractal/index.html
Students can experiment and create their own fractals using this online interactive tool, a prototype offered by the National Council of Teachers of Mathematics.

http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibpuzzles.html#bricks
Surrey University in the United Kingdom challenges students to "The Puzzling World of Fibonacci Numbers" by presenting a variety of puzzles, from easy to more challenging, that focus on Fibonacci's sequence.

http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html
Explore Fibonacci numbers and the Golden Section in nature through informative texts, graphics, and interactives. The site also includes an extensive list of suggested extension activities.

http://www.sci.mus.mn.us/sln/tf/books/naturesdesign.html
This Internet site, maintained by the Science Museum of Minnesota, contains activities and resources. In "By Nature's Design" students see photographs of patterns in nature - the beauty, symmetry, and diversity of nature's designs.

Print
Briggs, J. (1992). *Fractals: The Patterns of Chaos.* New York, NY: Touchstone.

Marzano, R. J., Pickering, D. J., & Pollock, J. E. (2001). *Classroom instruction that works: Research-based strategies for increasing student achievement.* Alexandria, VA: Association for Supervision and Curriculum Development.

Chessboard Patterns

NCTM Standards Addressed

Algebra

- **Standard:** Use mathematical models to represent and understand quantitative relationships
  - **Grade 9-12 Expectation:** Use symbolic expressions to represent relationships from various contexts.

Objective: Students observe the relationship between the size of squares and the number of different-sized squares on a chessboard. Then, students describe the pattern.

> **Teacher Tip**
> Emphasize that squares can overlap. For example the whole 8 x 8 chessboard is one square that overlaps 64 one-by-one squares. This is a difficult concept for students to understand.

Materials:

- Chessboard or graph paper so students can create an 8 x 8 checkerboard pattern of their own

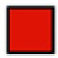
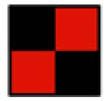
Procedure:

1. Show the clip "Mathematics in Games: Chessboard" from the *Donald in Mathmagic Land* DVD. This clip shows Donald Duck as he considers the mathematics involved in the game of chess. While students watch the clip, have them pay close attention to the chessboard.

2. Assemble students in small groups of three or four students per group for this exercise.

3. On the first activity screen after the clip, show students the 8 x 8 chessboard and ask them, "How many squares are there?" This will allow students to become engaged in the problem. At first they might say there are 64 squares. But then, someone will ask if the big square counts. All of a sudden, students will realize that it is a bigger question than they first realized. Their curiosity tends to get the best of them, and they rapidly become engaged in finding out how many squares. Ask them to count how many there are of each size square on the chessboard.

4. Do not show activity screen two to the students until they have had a chance to figure out the problem in their small groups. If students get stuck, activity screen two might help them organize their thinking by asking them to create a table. If you use activity screen two, explain to students that they can record their counts on a chart similar to this.

						
Size of square	1 x 1	2 x 2	3 x 3	4 x 4	5 x 5	6 x 6
Number of each size	64	49				

5. Activity screen three has some questions for students to answer. The questions center on patterns that students might see as they complete the chart. One pattern that is apparent from the chart on screen two is the number of 1 x 1 and 2 x 2 squares. Sixty-four and 49 are square numbers. Students might think that the number of 3 x 3 squares is 36. Once students have completed the chart, they should add up all of the square totals to answer the final question on activity screen three.

6. Ask students questions similar to the following:
   - Can you find a rule that will help you determine the number of squares of all sizes in a square grid of any size?
   - How many squares are in a smaller grid? (going smaller first might help students find a pattern)
   - How many squares in a 10 x 10 grid?
   - How many squares in a 50 x 50 grid?
   - How many squares in an n x n grid?

7. Conclude this lesson by having students develop a symbolic relationship, formula or statement to communicate the pattern that they discovered in the chessboard. Several possible student-generated statements may be: a) *The total number of squares on a chessboard is equal to the sum of the product of each number multiplied by itself from one to eight;* or b) *(9 – n) squared equals how many squares of size n x n can be found on the chessboard, where n is the dimension of the square.*

Resources:

http://mathforum.org/library/drmath/sets/select/dm_squares_checkerboard.html
This Web site contains a selection of answers from the Dr. Math archives to “How Many Squares in a Checkerboard?”

http://math.rice.edu~lanius/domath/page3a.html
This Web site contains an interactive chessboard.

Dilations

NCTM Standards Addressed

Geometry

- **Standard:** Apply transformations and use symmetry to analyze mathematical situations
  - **Grade 9-12 Expectation:** Understand dilations of objects by using sketches and coordinates

Objective: Students will identify scale factor of an object's dilation. Students will create a graph that shows a coordinate dilation with a certain scale factor.

Materials:

- Graph Paper
- Ruler

Procedure:

1. Show the clip "Mathematics in Games: Chessboard" from the *Donald in Mathmagic Land* DVD. This clip shows Donald Duck as he considers the mathematics involved in the game of chess. While students watch the clip, have them pay close attention to what happens to Donald (dressed as Alice) when he eats something from the chest.

2. Explain that when Donald doubled his size, this is considered a dilation. On the "Donald Dilations" activity screen, show how the distance from the center to the original Donald is equal to the distance from the original Donald to the dilation Donald, which indicates a scale factor of two since the scale is doubled.

3. On the "Coordinate Dilations" activity screen, students are shown an original and dilated triangle. Students should use the method described in the previous screen (or devise one of their own) to determine the scale factor of the triangles. Finally, students are instructed to create a coordinate dilation with a scale factor of three.

4. Conclude the lesson by asking students to think about how scale factors might be used in everyday life. Students might suggest scale models, maps, font sizes, enlarging photographs, or making enlargements on a photocopier.

Resources:

http://matti.usu.edu/nlvm/nav/frames_asid_166_g_2_t_3.html?open=activities
This online coordinate geoboard can be used to create dilations.

http://matti.usu.edu/nlvm/nav/frames_asid_296_g_4_t_3.html?open=activities
This virtual manipulative allows students to experiment with dilation transformations.

http://regentsprep.org/Regents/math/math-topic.cfm?TopicCode=dilate
http://regentsprep.org/Regents/math/math-topic.cfm?TopicCode=codilate
Visit these Web sites to access several helpful instructional resources on dilations provided by Oswego City School District.

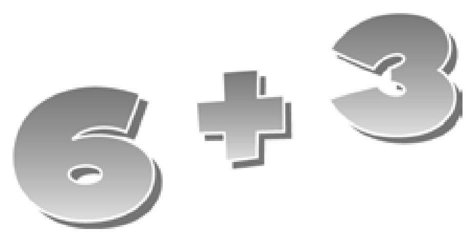
Math All Around Us

The following interdisciplinary activities can be adapted for all grade levels.

1) Students don't have to travel back three millennia like Donald does in order to develop an awareness of how math is used every day. Invite students to look around their school with mathematical eyes for a set period of time (a full day, an hour, a recess, or lunch break). Reproduce the chart on page 40 for students to list ways math is used during this time period. Let students share their lists in small groups. Then have each group vote for their most interesting or unusual findings and share them with the entire class.

2) Some jobs like accounting and bank telling require obvious math skills, but math is an important part in performing all jobs, even if it is used in less obvious ways. Lead the class in brainstorming careers that require math skills. Divide students into groups and allow each group to select a career or assign one of the following:

- Airline pilot
- Police officer
- School principal
- Heart surgeon
- Film producer
- Grocery store manager
- Stay-at-home parent
- Zookeeper
- Auto mechanic
- Building contractor



Have students discuss and record how people with that job might use mathematics while they work. Ask students to share their results with the class and encourage other students to add their own ideas.

Have students learn about a mathematician who shares their birthday. If you have access to the Internet, students can quickly find the names of more than one by visiting <http://www-groups.dcs.st-and.ac.uk/~history/Day_files/Year.html>. Have students learn some interesting facts about the mathematicians they choose and share those facts with the class – perhaps on their birthday!

3) In the video, Donald travels back to Pythagoras's time – about 530 B.C. Math and numbers existed in many other ancient societies, too. Assign groups of students one of the following numeral systems: Egyptian hieroglyphs; Babylonian cuneiform; Chinese rods; Mayan bars and dots; ancient Greek alphabet; Hindu-Arabic numerals; and Roman numerals. Have groups research the numeral system they've been assigned. They'll want to find out what symbols were used to represent numerals and learn to draw them, how each numeral system depicted numbers higher than 9, how they added and subtracted, and whether the system continues today. Have each group teach their classmates about their number system.

4) In the video, Donald learns that sometimes math is all fun and games. Invite students to demonstrate a card trick, magic trick, or mind teaser that uses math in some way.

Students may want to look through books to get ideas, or if they have access to the Internet they can visit <http://www.cut-the-knot.org/games.shtml>. Have students try to guess the math principle(s) involved in each trick and let the demonstrator explain if no one guesses correctly.

5) The video shows how the concept of the Golden Rectangle influenced architecture through the ages. Explain to students that people around the world design homes and buildings with shapes they find visually pleasing and practical for their needs. Have students design houses or buildings that are visually pleasing to them and make a model of them out of empty boxes, heavy paper, and other materials. You might want to provide pictures of a variety of structures, such as a conical tipi, the Pentagon in Washington, D.C., a Kenyan round house, a medieval European castle, and an Egyptian pyramid to generate ideas. Display the models in your classroom and provide time for students to explain the mathematical principles behind their designs.

6) Have students interview a professional to find out how he or she uses math on the job. Students may choose family members, friends, neighbors, or other members of the local community. Ideally, students will visit their subject's place of employment to see for themselves how math is used, but home or telephone interviews are acceptable as well. Students should ask questions such as, "How do you use math in your job?" "How often do you use math?" Provide time for students to share what they learned in small groups, and ask each group to summarize their findings and share them with the entire class.

7) Bring in a weekend edition of your local newspaper and give groups of students a section, such as World News, Local News, Weather, Sports, Business, Travel, Classifieds, and so on. If your local paper is too small to divide among all of the groups, provide a larger metropolitan paper instead. Instruct each group to analyze their section of the newspaper and circle anything in the layout or articles they find that pertains to math.

8) Mathematical thinking has opened many doors to the exciting adventures of science and invention. Have students choose a scientific invention such as the telescope, and find out the math that was involved as a basis for the invention's discovery.

9) Have students attend a game at school or in their neighborhood, or view one on television, and observe the event from a mathematical perspective. Some aspects of the game you might encourage students to analyze include the geometric makeup of where the game is played; the skill required to play the game such as logic, power, or finesse; how the game is measured or scored; and how statistics are calculated. Invite students to share their results with the class.

10) Invite students to create a Web page or book that is a collection of situations that teenagers typically find themselves in with descriptions of how math is useful in each. Have students submit their scenario ideas to you first in order to ensure that each student contributes an original topic. Scenarios can be creative and humorous as long as the math connections make logical sense. Students with access to the Internet can visit the "What Good is Math?" site at <http://oncampus.richmond.edu/academics/education/projects/webunits/math/home.htm> if they need ideas to get started, or you can write these ideas on the blackboard:

- Shopping at the mall
- Buying a car
- Eating lunch
- Planning a party
- Being treasurer of a club
- Playing in the school band

11) Because *Donald in Mathmagic Land* was produced before the advent of personal computers, there is no mention of their important contribution to society. Have students research and make a book, video, or multimedia presentation in the style of *Donald in Mathmagic Land* that shows the history of computers. Have students start with the earliest computers and the inventors responsible for their creation and move forward to the personal computers, modems, and other computer technology available today. Students will also want to record some of the things people are able to do now because of this technology that couldn't be done fifty years ago.

12) The tangram puzzle is an excellent example of a game that applies math logic. In this ancient Chinese game, seven geometric pieces are used to create designs. Find reproducible tangram puzzle pieces in a book or at the Strong Museum Internet site at <http://www.strongmuseum.org/kids/tangram.html>. Reproduce the pieces onto brightly colored paper and have students try to use all seven pieces (a square, a rhomboid, and five triangles) to create their tangram designs. You might suggest they try to make a chair, a shirt, a rooster, or a sailboat. Answers to these tangram puzzles and additional ideas can also be found at the Strong Museum Internet site. An interactive tangram puzzle is also available at <http://matti.usu.edu/nlvm/nav/frames_asid_268_g_1_t_3.html?open=activities>.

13) Donald learns that there are no limits to what the imagination can come up with. A great number of the world's inventions are the result of a person solving a problem in an imaginative way. Place a variety of items on a table somewhere in your classroom. Provide both practical and seemingly impractical things like duct tape, ball bearings, Popsicle sticks, rubber bands, a pencil, marshmallows, and so on. Tell students who are interested that they can invent anything they want using at least one of those items, from the serious to the whimsical, but the invention should be functional. Display finished inventions and have students discuss what their inventions are and some of the challenges they encountered while making them.

EXPANDED TIMELINE

Donald goes back in time and learns that Pythagoras and his followers shared their ideas in secret. Divide students into small groups and ask each group to choose a time span or event listed in the expanded timeline, found in the ROM menu on the DVD. Ask students to become experts on the event that they chose. Each group should research the event and present their findings through a product of their choice. Examples might include: poster, multimedia presentation, or other creative visual project.

On the DVD, try playing the Interactive Timeline game. Pick either People or Inventions, and put them in the right order.

Resources

WHO'S WHO IN MATHEMATICS

If you have access to the Internet and want to read full biographies of these mathematicians or choose from over a thousand others, visit <http://www-groups.dcs.st-andrews.ac.uk/%7Ehistory/BiogIndex.html>.

Pythagoras of Samos (c. 580 B.C. – c. 500 B.C.)
Pythagoras was a Greek philosopher and mathematician who left Samos in 532 B.C. and founded a school of philosophy and religion in Italy. The school made significant contributions to music theory, astronomy, and mathematics. The Pythagorean Theorem, a squared + b squared = c squared, relates the three sides of a right triangle.

Archimedes of Syracuse (c. 287 B.C. – c. 212 B.C.)
Archimedes was a Greek mathematician, scientist, and inventor. He formulated the principles of buoyancy after he stepped into his bath one day and noticed the displacement of water. According to Plutarch, Archimedes was killed during a Roman invasion of Syracuse. Archimedes was studying a math problem and told a Roman soldier he wouldn't leave his home until he had finished it. The angry soldier replied by drawing his sword and killing him.

Hypatia of Alexandria (c. 370 – c. 415)
Hypatia was the first known female mathematician. Home-schooled by her father, Hypatia herself became an acclaimed math, philosophy, and astronomy teacher in Alexandria, Egypt. Hypatia edited *On the Conics of Apollonius*, a work that later developed into concepts of hyperbolas, parabolas, and ellipses. At age 45, Hypatia was killed by a mob during a period of religious and civil unrest.

Leonardo Fibonacci of Pisa (c. 1170 – c. 1240)
Fibonacci was an Italian mathematician. He was so interested in the mathematical systems he saw while traveling in foreign countries that in 1202, Fibonacci published *Liber Abaci*, Latin for "The Book of Abacus." This book introduced Arabic numerals and the Hindu-Arabic decimal system into Europe. He discovered the curious sequence: 1, 1, 2, 3, 5, 8, 13, and so forth in which each number is the sum of the two previous numbers, and the ratio of any term to its predecessor gets closer to the Golden Ratio the further out you go.

Blaise Pascal (1623 – 1662)
Blaise Pascal was a French mathematician, inventor, physicist, and theologian. He invented the barometer and the hydraulic press, and in correspondence with fellow mathematician Pierre de Fermat, he set the stage for the theory of probability. Pascal died at age 39 of what was probably stomach cancer.

Isaac Newton (1642 – 1727)
Isaac Newton was an English mathematician and scientist. While homebound during the Bubonic Plague, Newton invented calculus. He later developed influential theories about light, laws of motion, and gravity, although it is doubtful he discovered the principle of gravity after watching an apple fall. During his lifetime he was a Cambridge professor, a Member of Parliament, Master of the Mint, and was knighted in 1705.

Leonhard Euler (1707 – 1783)
Leonhard Euler was perhaps the most prolific mathematics writer of all time. Euler was born in Switzerland but lived most of his life in St. Petersburg, Russia. He wrote more than 800 books and papers on math, astronomy, and physics, almost half of them after he had gone blind. Many of his notations, such as e and π, are still used today.

Charles Lutwidge Dodgson (Lewis Carroll) (1832 – 1898)
Charles Lutwidge Dodgson, better known by his pseudonym Lewis Carroll, was born in England and was a mathematics lecturer at Oxford and a deacon in the Anglican Church. He wrote several mathematics books, but is famous for his books *Alice's Adventures in Wonderland* and *Through the Looking Glass*.

Albert Einstein (1879 – 1955)
Albert Einstein was born in Germany. His teachers doubted he would ever learn mathematics because he had so much trouble calculating sums. Einstein became a lecturer and professor, and won the Nobel Prize for Physics in 1921. Einstein moved to the United States in 1933 after the Nazis forced him from his home and his teaching job in Berlin. He became a professor at Princeton University. His equation $E = MC^2$ led to the evolution of nuclear fission and the atomic bomb. In 1952 Einstein declined an offer to serve as Israel's president.

George Polya (1887 – 1985)
George Polya was a Hungarian mathematician who later moved to the United States. He made contributions to probability, number theory, and geometry. Polya was an inspiration to mathematics teachers by demonstrating the importance of learning how to solve problems creatively instead of how to do arithmetic slavishly. In his book *How to Solve It*, Polya wrote: "If you can not solve the proposed problem try to solve first some related problem."

Glossary Term and Definition
Pi –Pi (π) represents the ratio of the circumference of a circle to its diameter (22/7 which is approximately 3.14).
Pentagram – A pentagram is a five-pointed star. Formed by five straight lines, a pentagram connects the vertices of a pentagon and encloses another pentagon in the complete figure.
Golden Section – The Golden Section refers to a ratio between two dimensions of a plane figure, observed especially in art. When a line is divided into two segments, the ratio of the smaller line to the larger line is roughly three to five.
Infinity – Infinity is an immeasurably large amount that increases indefinitely (x+1) and has no boundaries or limits.
Proportion – Proportion is a statement that two ratios are equal.
Pentagon – A pentagon is a polygon with five sides.
Geometrical (figures) – Figures made up of points, planes, line segments, lines, rays, or angles are described as geometrical shapes.
Angle – An angle consists of two rays with a common end point.
Sphere – A sphere is the set of all points in space at a given distance from a given point called the center.
Cone – A cone is a pyramid-like object with a circle-shaped base.

BIBLIOGRAPHY

If you liked the topics in *Donald in Mathmagic Land*, here are some books about math appreciation, the history of math, and using math in fun and creative ways that you might also enjoy:

The Billiard Congress of America. *Billiards: The Official Rules and Records Book.* New York: Lyons & Burford, 1992.

Cole, K.C. *The Universe and the Teacup: The Mathematics of Truth and Beauty.* New York: Harcourt Brace & Company, 1998.

Jacobs, Harold R. *Mathematics: A Human Endeavor.* New York: W.H. Freeman & Company, 1970.

Katz, Victor. *A History of Mathematics: An Introduction.* New York: HarperCollins College Publishers, 1993.

Muschla, Judith A., and Gary Robert Muschla. *The Math Teacher's Book of Lists.* Englewood Cliffs, New Jersey: Prentice Hall, 1995.

Ohanian, Susan. *Garbage Pizza, Patchwork Quilts, and Math Magic.* New York: W.H. Freeman, 1993.

Stewart, Ian. *The Magical Maze: Seeing the World Through Mathematical Eyes.* New York: John Wiley & Sons, Inc., 1997.

Thiessen, Diane, and Margaret Matthias. *The Wonderful World of Mathematics: A Critically Annotated List of Children's Books in Mathematics.* Reston, Virginia: National Council of Teachers of Mathematics, 1992.

Wakeling, Edward. *Lewis Carroll's Games and Puzzles.* New York: Dover Publications, 1992.

Math Around Us

Numbers and Counting

Measurement

Geometry

Probability and Statistics

Fractions

Patterns